

# APPLICATION OF BAYESIAN PROBABILISTIC NETWORKS FOR LIQUEFACTION OF SOIL

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## Abstract

The paper considers the application of Bayesian probabilistic networks (BPN) in liquefaction analysis. BPN's facilitate risk assessment in a generic framework by using indicators to relate the generic representation to the specific condition causing liquefaction. A basic introduction to BPN is provided and the concept of indicators is applied in accordance with potential site specific information. The methodology is then applied to a site located in Adapazari, Turkey, where extensive liquefaction problems occurred during the catastrophic 1999 Kocaeli earthquake. First the indicators related to soil liquefaction including their probabilistic modelling are summarized. For the liquefaction analysis state-of-the-art procedures are applied. The example illustrates how BPN's can be used to assess the probability of liquefaction.

## Keywords

Liquefaction, Bayesian probabilistic networks, uncertainty.

## 1 Introduction

Earthquake risk management constitutes a very complex problem framework. It requires a realistic and reliable modelling of the seismic hazard, the soil response including soil failures like liquefaction, the structural response, the damage assessment and all possible consequences. The tools for the structuring and modelling of such large complex problems are fault and event trees. The entire decision problem structure can be modelled by these two supplementary analysis tools. The main drawback of these tools however is the exponentially growing size of the branches of the trees with increasing number of variables. This makes the model awkward and very difficult to communicate to third parties for validation purposes. Bayesian probabilistic networks (BPN's) provide a remedy for these drawbacks, as they map the problem framework by a graphical representation of nodes and directed links explicitly showing the probabilistic dependence between the nodes and the information flow in the model; nodes characterising the uncertain quantities of the problem, arrows the causal interrelation between these quantities. Conditional dependencies and new evidence may easily included.

Bayesian probabilistic networks, also called belief networks or probabilistic causal networks, have become popular during the last two decades in the research areas of artificial intelligence, probability assessment and uncertainty modelling (Pearl 1988). The ideas and techniques have gained recognition also in other engineering disciplines and natural sciences, especially in problems involving high complexities and uncertainties, see also

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Faber et al. (2002). The description and assessment of natural hazards and the quantification of their related risks appears to be a problem for which BPN's can be a helpful tool. In Antonucci et al. (2004) BPN's are applied to assess hazards due to debris flow, in Straub (2005) to natural hazards risk assessment and in Bayraktarli et al. (2005) to earthquake risk management.

A key element in the pursued approach is the quantification of the effect of various types of observable information (condition indicators) on the elements of the functional chain of an earthquake. These condition indicators have very different characteristics and necessitate integrating different expertise into the project. Therefore an interdisciplinary research program "Management of Earthquake Risks using Condition Indicators" (MERCI, 2004), funded by the Swiss National Science Foundation, is currently working on generic models for the management of earthquake risks. These models require also a generic handling of the soil response during an earthquake. In the present paper the evaluation of one type of soil response, the earthquake induced liquefaction, by a BPN will be discussed.

During an earthquake, loosely packed water-saturated sediments near the ground surface may lose their strength and stiffness as a result of pore water pressure increase. This phenomenon, known as liquefaction may cause serious damage to the built environment as experienced during the earthquakes in Niigata (1964), Loma Prieta (1989) and Kocaeli (1999) (Geoengineer website, 2006). The soil liquefaction is quantified using deterministic and probabilistic techniques either based on laboratory tests or empirical correlations of in-situ index tests with field case performance data. The deterministic empirical correlation in using Standard Penetration Test (SPT) proposed by Seed and Idriss (1971) is widely used in practice to evaluate the potential for soil liquefaction. A revised version of this so-called simplified procedure (Youd et al. 2001) will be applied in this paper to illustrate the application of BPN in liquefaction analysis.

## 2 Bayesian probabilistic networks for risk assessment

BPN's constitute a flexible, intuitive and strong model framework for Bayesian probabilistic analysis (Jensen 2001). BPN's may substitute both fault and event trees and can be used at any stage of a risk analysis. Due to their mind mapping characteristic, they comprise a significant support in the early phases of a risk analysis, where the main task is to identify the potential hazard scenarios and the interrelation of events leading to adverse events. BPN's provide a strong tool for decision analysis, including prior analysis, posterior analysis and pre-posterior analysis. Furthermore, they also provide a tool for diagnosing systems, i.e. identifying the event scenarios, which with the largest likelihood lead to specific adverse events of interest.

$N$  is a Bayesian network triplet  $(V, A, P)$ , where

- $V$  is a set of variables  $v_i, i = 1, 2, 3, \dots$
- $A$  is a set of links showing causal interrelations between the variables. The links  $A$  and the variable set  $V$  constitute a directed acyclic graph.
- $P = \{P(v | \pi_v) : v \in V\}$ , where  $\pi_v$  stands for the set of parents of  $v$ . In words  $P$  is the set the conditional probabilities of the all variables given their parents.

It is common to visualise the variables in a BPN as nodes. Two additional elements, decision nodes and utility nodes, may be added to a BPN enabling the BPN to solve decision problems. Such BPN's are also known as influence diagrams. A decision node denoted by a rectangle shows the alternative actions to be chosen by the decision maker and utility nodes visualised by diamonds show the consequences of the chosen action. A BPN may be formulated by the following steps, see also Figure 1:

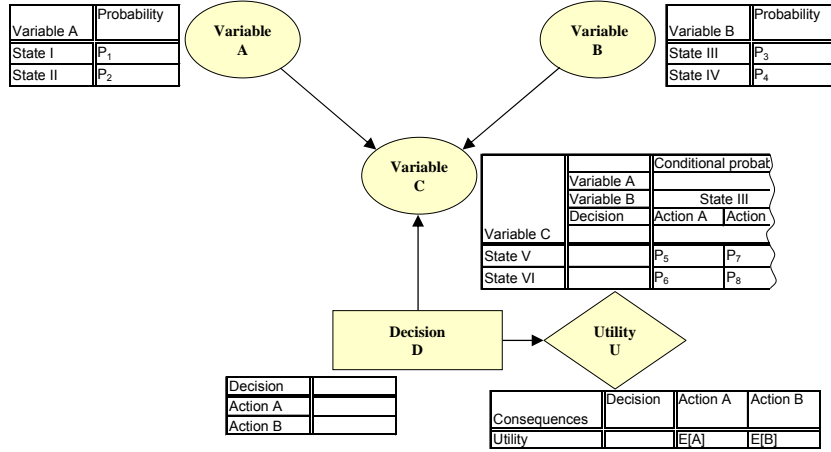


Figure 1. A generic BPN with states and probability tables

- Variables necessary and sufficient to model the problem framework of interest are identified.
- Causal interrelations existing between the nodes are formulated. They are graphically shown in terms of arrows connecting variables.
- A number of discrete mutually exclusive states are assigned to each variable.
- Probability tables are assigned for the states of each of the variables.

More formally, the BPN maps the joint probability distribution  $P_N(V)$  of a considered system.

$$P_N(V) = \prod_{v \in V} P(v | \pi_v) \quad (1)$$

As an example, for the generic BPN in Figure 1 the joint probability is given by:

$$P_N(A, B, C, D, U) = P(A)P(B)P(C | A, B, D) \quad (2)$$

The marginal probability of any variable, say variable C in Figure 1, is defined by marginalising all variables different from variable C out of the joint probability:

$$P_N(C) = \sum_{V/C} P_N(V) \quad (3)$$

There may be evidence that some of the variables have specific values. For example, the variable B in the BPN in Figure 1 may be observed to be in State III. Then the posterior probability of any variable in the BPN, for example of variable C is defined as:

$$P_N(C | B = \text{State III}) = \frac{P_N(C, B = \text{State III})}{P_N(B = \text{State III})} \quad (4)$$

Very efficient so-called inference engines are available that makes the calculations of Eq.'s (2) - (4) tractable (Jensen 2001).

### 3 Example Application

For the example application, a site in the city centre of Adapazari/Turkey, which was affected by the 17th August, 1999 Kocaeli  $M_w$  7.4 earthquake is chosen. Many buildings in Adapazari suffered damage due to liquefaction induced ground settlement during that earthquake. The soil profile used in the following analyses is taken from PEER (2000). Figure 2 illustrates the soil profile used in the analyses.

Depth (m)	Layer	$N_{(1,60)}$ -Value	Soil type	Description
0.0				
1.0	1	n.a	fill	fill
1.65	2	8	ML-CL	silty clay
2.4	3	10	ML	silt
3.25	4	n.a	CH-MH	high plasticity clay-elastic silt
4.2	5	8	CL-CH	lean clay-fat clay
4.7	6	16	ML	silt
5.75	7	n.a	CL	clayey silt
6.7	8	10	MH-CH	elastic silt-high plasticity clay
7.9	9	19	ML	silt
9.5	10	41	SP-SM	poorly graded sand-silty sand
10.4	11	25	SP-SM	poorly graded sand-silty sand

Figure 2. Soil profile of SPT A-2 considered in the example application (PEER, 2000)

### 3.1 Procedure for soil liquefaction analysis

As indicated in the introduction a revised version of the so-called simplified procedure will be used for the liquefaction analysis (Youd et al. 2001). Soil types expected to exhibit no susceptibility for liquefaction due to their high fines content and liquid limit are excluded from the analysis using the “Modified Chinese Criteria” (Andrews and Martin 2000), see Table 1.

Table 1. Liquefaction susceptibility according to the “Modified Chinese Criteria”

	Liquid Limit, LL<32	Liquid Limit, LL≥32
Fines Content <10%	Susceptible to liquefaction	Further studies required
Fines Content ≥10%	Further studies required	Not susceptible to liquefaction

The limit state function for liquefaction is given by:

$$g(x) = CRR_{7.5} \cdot MSF \cdot K_{\sigma} - CSR = 0 \quad (5)$$

where  $CRR_{7.5}$  is the cyclic resistance ratio for earthquake magnitudes of about 7.5,  $CSR$  is the cyclic stress ratio,  $MSF$  is the earthquake magnitude scaling factor to correct for moment magnitudes  $M_w$  smaller or larger than 7.5,  $K_{\sigma}$  is a correction factor to extrapolate the procedure to layers with overburden pressures larger than 100kPa (Youd et al. 2001). The cyclic resistance ratio and cyclic stress ratio is calculated by:

$$CRR_{7.5} = \frac{1}{34 - (N_1)_{60}} + \frac{(N_1)_{60}}{135} + \frac{50}{[10 \cdot (N_1)_{60} + 45]^2} - \frac{1}{200} \quad (6)$$

where  $(N_1)_{60}$  is the SPT blow count normalized to an overburden pressure of 100 kPa, a hammer efficiency of 60% and borehole diameter of 65-115mm. Eq. (6) is valid for  $(N_1)_{60}$  values less than 30. For larger values clean granular soils are classified as non-liquefiable. The cyclic stress ratio is calculated by:

$$CSR = 0.65 \cdot \frac{a_{\max}}{g} \cdot \frac{\sigma_{vo}}{\sigma'_{vo}} \cdot r_d \quad (7)$$

where  $a_{\max}$  is the maximum acceleration in the soil layer,  $\sigma_{vo}$  and  $\sigma'_{vo}$  are the total and effective stresses in the middle of the soil layer, and  $r_d$  is a depth reduction coefficient in accordance with Seed and Idriss (1971).

To implement this procedure into a Bayesian probabilistic network a generic and probabilistic approach is required. The soil profile of SPT A-2 is used for illustrating the procedure. The density, void ratio, water content and layer thickness for the calculation of the total and effective stresses are assumed to be random variables, whereas the remaining parameters are treated deterministically.

A set of acceleration time histories is generated for the soil response analyses. For this purpose the attenuation relationship proposed by Boore et al. (1997) for estimating the pseudo-acceleration response spectra for the random horizontal component at 5% damping is used. 16 pairs of moment magnitudes  $M_w$  (5.5, 6.5, 7.0, 7.5), epicentre distances  $R$  (10, 20, 40, 80 km) and site class (rock) are selected for the estimation of the pseudo-acceleration response spectra. Using a modified version of SIMQKE (Gasparini and Vanmarcke 1976) by Lestuzzi (2000) 10 samples of accelerogram time histories for each pair of these are generated, resulting in 160 acceleration time histories.

The 160 acceleration time histories are applied at the bedrock level and propagated vertically through the soil layers in the 1D analysis program Shake (Schnabel et al. 1972). Using equivalent linear soil properties in an iterative procedure, the soil properties compatible with the strains and the acceleration time histories in each layer are calculated. The maximum accelerations in each layer are then used in the liquefaction analysis.

### 3.2 Probabilistic modelling of soil parameters

Sources and types of uncertainty in geotechnical engineering can formally be grouped into aleatory and epistemic uncertainty (Jones et al. 2002). Aleatory uncertainty represents the inherent randomness of the soil properties depending on the spatial variability of the property. Epistemic uncertainty represents the erroneous modelling, the lack of information and shortcomings in the measurements. The uncertainty from the spatial variability of soil parameters in natural soil deposits is not explicitly considered in the present paper; only the variation of the layers thicknesses in the considered soil profile is taken into account.

For the present analyses the uncertainties in the following parameters are considered: density, water content, void ratio and layer thickness. For all soil types these random variables are assumed to be normally distributed (Lacasse & Nadim 1996). The parameters of the distribution are taken from the Swiss Standard SN 670 010b (VSS 1999) published by the Association of Swiss Road and Traffic Engineers, where a set of 6000 laboratory tests were analyzed and the parameters for each soil type were classified after the Unified Soil Classification System. The parameters used in the example application are given in Table 2. For the layer thickness the values given in the boring logs are taken as mean values with a coefficient of variation of 15%.

Table 2. Uncertainty of soil parameters used in the analyses

Soil Type	Density [ $t/m^3$ ]		Water content [%]		Void Ratio [-]	
	Mean	Stdv	Mean	Stdv	Mean	Stdv
A	2.04	0.15	5.5	2.5	0.26	0.05
SP-SM	2.03	0.18	33	9.3	0.60	0.25
ML	1.99	0.20	36	19.7	0.77	0.51
ML-CL	2.11	0.11	37	5.3	0.55	0.15
CL	2.13	0.10	44	5.4	0.55	0.14
MH-CH	1.76	0.26	44	29.3	1.33	0.63
CL-CH	2.02	0.24	37	16.6	0.80	0.43

### 3.3 Estimation of probability of liquefaction

The limit state function in Eq. (5) is evaluated to obtain the probability of liquefaction by Monte Carlo simulations ( $N=10'000 \div 10'000'000$ , depending on the probability of liquefaction in the layer) using the uncorrelated normal distributed random variables; layer thickness, density, water content and void ratio. The maximum acceleration  $a_{max}$  values for each layer are taken from the one dimensional soil response analysis. The results are given in Table 3.

Table 3. Probabilities of liquefaction for each layer and for each pair of  $M_w$ ,  $R$

$M_w$	R [km]	Probability of liquefaction for layer $i$ [%]										
		1	2	3	4	5	6	7	8	9	10	11
5.5	10	n.a.	0.00	0.02	n.a.	0.03	0.02	n.a.	0.04	0.04	n.liq.	0.04
	20	n.a.	0.00	0.00	n.a.	0.02	0.01	n.a.	0.01	0.03	n.liq.	0.02
	40	n.a.	0.00	0.00	n.a.	0.01	0.01	n.a.	0.01	0.02	n.liq.	0.02
	80	n.a.	0.00	0.00	n.a.	0.01	0.01	n.a.	0.01	0.01	n.liq.	0.02
6.5	10	n.a.	0.00	0.03	n.a.	14.80	0.05	n.a.	99.95	0.07	n.liq.	0.08
	20	n.a.	0.00	0.02	n.a.	0.17	0.03	n.a.	0.09	0.06	n.liq.	0.05
	40	n.a.	0.00	0.01	n.a.	0.05	0.02	n.a.	0.05	0.03	n.liq.	0.04
	80	n.a.	0.00	0.01	n.a.	0.02	0.02	n.a.	0.02	0.03	n.liq.	0.03
7.0	10	n.a.	11.36	2.46	n.a.	99.93	0.12	n.a.	99.94	0.13	n.liq.	8.85
	20	n.a.	0.22	0.08	n.a.	98.64	0.04	n.a.	99.86	0.06	n.liq.	0.06
	40	n.a.	0.00	0.01	n.a.	0.10	0.03	n.a.	0.11	0.05	n.liq.	0.05
	80	n.a.	0.00	0.01	n.a.	0.04	0.02	n.a.	0.04	0.04	n.liq.	0.03
7.5	10	n.a.	99.95	99.97	n.a.	99.93	99.93	n.a.	99.94	99.95	n.liq.	99.22
	20	n.a.	8.10	3.88	n.a.	99.93	0.09	n.a.	99.95	0.11	n.liq.	0.11
	40	n.a.	0.00	0.02	n.a.	5.31	0.04	n.a.	60.35	0.06	n.liq.	0.06
	80	n.a.	0.00	0.02	n.a.	0.10	0.03	n.a.	0.05	0.03	n.liq.	0.03

\* n.a.: SPT number of blows were not available, n.liq.: not liquefiable, since  $(N_1)_{60} > 30$

### 3.4 Bayesian probabilistic network analysis

Figure 3 illustrates the BPN considered for the example application in the present paper. The annual probabilities for each moment magnitude  $M_w$  ( $M_w = 5.5, 6.5, 7.0, 7.5$ ) are calculated using the Gutenberg-Richter magnitude recurrence relationship (Gutenberg and Richter, 1944) with recurrence rate parameters corresponding to Anatolian Trough source zone from Erdik et al. (1985). The seismic source zone is considered as a point source and the occurrence of strong earthquakes is assumed to follow a stationary Poisson process. These probabilities form the probability tables for 'Earthquake magnitude'. The epicentre distances are assumed to be  $R=10$  km, 20 km, 40 km and 80 km to the site and are presented by 'Earthquake distance'. 'Soil type' constitutes the states rock, gravel, sand, silt and clay. 'Spectral acceleration' is conditioned on these three nodes and on 'Soil response'. 'Soil profile' constitutes the different layers of the considered bore profile. 'PGA (Peak Ground Acceleration)' is assumed to be independent of 'Spectral acceleration' but dependent on the 'Earthquake magnitude', 'Earthquake distance', 'Soil type' and 'Soil profile'. The 'Modified Chinese Criteria' is implemented as a logical connection in 'Liquefaction susceptibility'. Its conditional probability table depends on 'Liquid limit (LL)' with the two states  $LL < 32$ ,  $LL \geq 32$  and on 'Fines content (FC)' with the two states  $FC < 10\%$ ,  $FC \geq 10\%$ . The node 'Liquefaction' comprises the conditional probabilities for liquefaction computed based on the revised version of the simplified procedure from Table 3, whereby the states of 'PGA' are taken from the simulated acceleration time histories. 'Soil response' has two states 'Ground amplification' and 'Liquefaction'. Conditional on the 'Spectral acceleration' and 'Soil

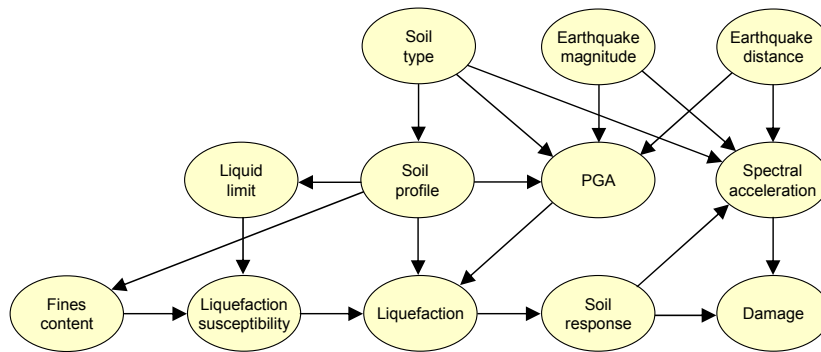


Figure 3. Bayesian probabilistic network for assessing liquefaction of soil

response', the probabilities of being in a predefined damage state form the conditional probability tables in the 'Damage' node. The conditional probability tables of the nodes 'Spectral acceleration' and 'Damage' are taken from Bayraktarli et al. (2005), where the BPN for the decision problem of retrofitting or not retrofitting the specified structures is discussed. For the BPN analysis the software package Hugin (Hugin 2005) is used. The analysis is performed for each layer of the considered soil profile resulting in the probability of liquefaction in the node 'Liquefaction'. The information flows from this node to the distribution of damage on the structure on the site. With observations of one or more of the variables the updated probabilities can be calculated easily using the BPN. As an example, for different  $M_w$ 's the probabilities of liquefaction for the soil layers calculated by the BPN are given in Figure 4. The sharp change of probability of liquefaction in one layer from one  $M_w$  to the next indicates that the critical PGA's are exceeded for in that range of  $M_w$ .

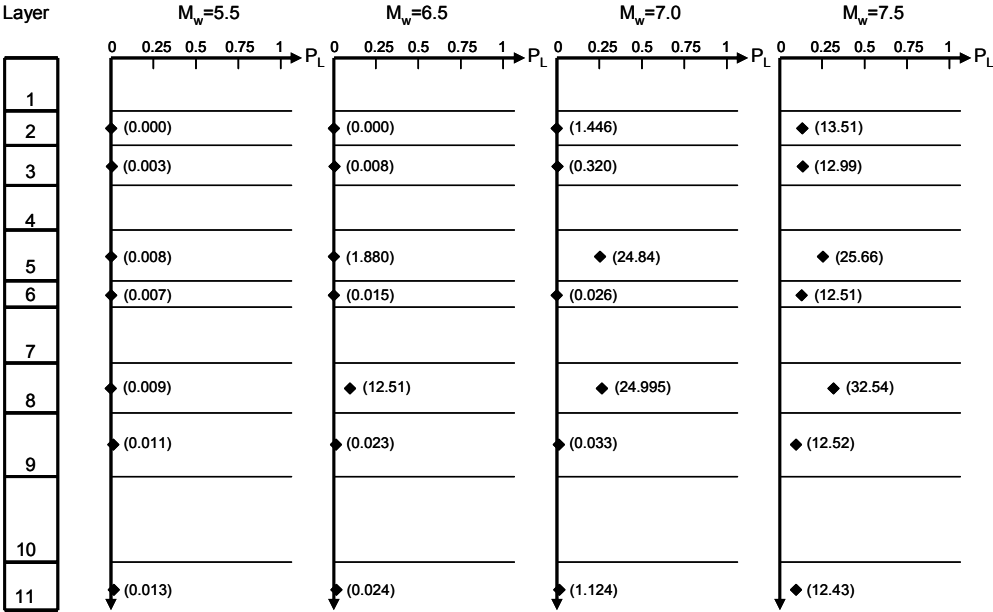


Figure 4. Probability of liquefaction for different  $M_w$  analysed using the BPN

### 4 Discussion

An approach for the evaluation of soil liquefaction by using a Bayesian probabilistic network (BPN) is considered. The evaluation of a BPN for liquefaction analysis of a site affected by the 17th August, 1999 Kocaeli  $M_w7.4$  earthquake is illustrated in an example. The state-of-the-art procedure, a revised version (Youd et al. 2001) of the simplified procedure by Seed and Idriss (1971) for the calculation of the probability of liquefaction, is implemented into a BPN. The uncertainties of the soil parameters density, water content, void ratio and layer thickness are taken into account. Probabilities of liquefaction are calculated using Monte Carlo simulations.

In the further work, the spatial variability of the soil parameters and modelling uncertainties of the procedures should be considered. Considering spatial variability will give a better estimate for the extent of the liquefaction event. Implementing modelling uncertainties will enable to track the sensitivity of optimal decision in regard to model choices. Furthermore, the correlation structure of PGA and spectral acceleration, the first required for the liquefaction analysis, the second for the structural analysis, will be determined.

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